B.A./B.Sc. 1st Semester (General) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMG1CC-1A/GE-1

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Show that $\lim_{x \to 0} x \cos\frac{1}{x} = 0$.

(b) Discuss about the continuity at $x = 0$ for the function $f(x) = [x] + [-x]$.

(c) Is Rolle's theorem applicable to the function $f(x) = \tan x$ in $[0, \pi]$? Justify your answer.

(d) Give the geometrical interpretation of Lagrange's Mean Value Theorem.

(e) Find the maximum value of $f(x) = 2x^3 - 6x^2$ in $[1, 3]$.

(f) If $y = \cos(m\sin^{-1}x)$, show that $(1 - x^2)y_1^2 = m^2(1 - y^2)$, where $y_1 = \frac{dy}{dx}$.

(g) Find the curvature of $r = a\cos\theta$ at $(0, {}^{\pi}/_{2})$.

(h) Find the asymptotes of the curve $xy^2 - x^2y = x + y + 1$.

(i) Is the curve $x^{2/3} + y^{2/3} = a^{2/3}$ symmetrical about both the co-ordinate axes? Justify your answer.

(j) Evaluate $\lim_{x \to 0}(\cos x)^{\frac{1}{x^2}}$.

(k) If $u = f\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

(l) Prove that the radius of curvature of a circle of radius 'a' is an invariant.

(m) Find the envelope of the straight line $y = mx + \frac{a}{m}$, m being the variable parameter $(m \neq 0)$.

(n) Mention an asymptote of the curve $xy = 1$.

(o) Prove that $\tan x > x$ whenever $0 < x < \frac{\pi}{2}$.

2. Answer *any four* questions from the following: 5×4=20

(a) If $V = \sin^{-1}\frac{x^2+y^2}{x+y}$, then prove that $xV_x + yV_y = \tan V$.

(b) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

(c) For what values of a and b the function

$$f(x) = ax + b, x \leq -1$$
$$= ax^3 + x + 2b, x > -1$$

be differentiable for all values of x? Hence find $f'(x)$. 3+2

(d) Use Taylor's theorem to prove that

$$x - \frac{x^3}{6} < \sin x < x \text{ for } 0 < x < \pi.$$

(e) Find the radius of curvature of $y = xe^{-x}$ at a point where y is maximum.

(f) Expand $\log(1 + \tan x)$ using Maclaurin's theorem.

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Find the asymptotes of the curve

$$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0.$$

(ii) Find the envelope of the family of parabolas $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$, where $a + b = c$, a, b being parameters. 5+5=10

(b) (i) State and prove Euler's theorem for a homogeneous function of two variables.

(ii) Trace the curve $x(x^2 + y^2) = a(x^2 - y^2)$, $a > 0$. (2+3)+5=10

(c) (i) Determine the position and nature of the double points on the curve

$$y(y - 6) = x^2(x - 2)^3 - 9.$$

(ii) If $f(x)$ be continuous in $a \leq x \leq b$ and $f'(x) > 0$ in $a \leq x \leq b$, prove that $f(x)$ is a strictly increasing function in $a \leq x \leq b$. 5+5=10

(d) (i) Examine whether $x^{\frac{1}{x}}$ possesses a maximum or a minimum and determine the same.

(ii) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$ 5+5=10